What you'll Learn About

- Average Rates of Change
- A Definition of the Derivative

An object dropped from rest from the top of a tall building falls $\boxed{y = 16t^2}$ feet in the first t seconds. Find the **average speed/average rate of change** during the first 2 seconds of flight.

$\frac{\Delta y}{\Delta x}$ Difference Quotient

$\boxed{\frac{64-0}{2-0}} = \frac{64}{2}\ \frac{\text{feet}}{\text{sec}} = 32 \frac{\text{ft}}{\text{sec}}$

$\boxed{(0, 0) \quad (2, 64)}$
sec ft

Find the average rate of change of $f(x) = \sqrt{4x+1}$ over each interval

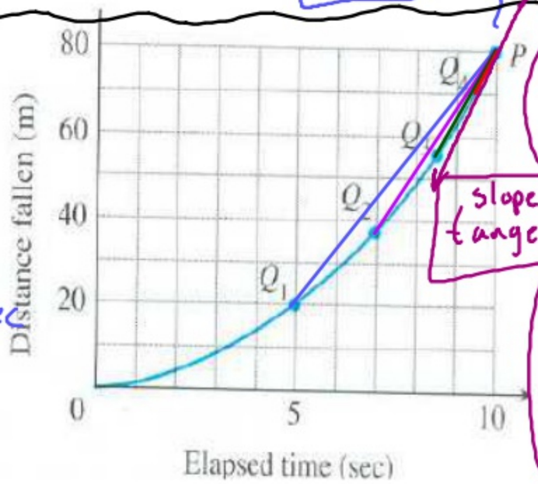
slope $= \frac{\Delta y}{\Delta x}$

$\sqrt{x^2} = \sqrt{4}$
$x = \pm 2$

a) [0, 2]
$(0, 1)$
$(2, 3)$ AROC $= \boxed{\frac{3-1}{2-0}} = 1$

b) [10, 12]
$(10, \sqrt{41})$
$(12, 7)$
$\boxed{\frac{7-\sqrt{41}}{12-10}}$
$\boxed{\frac{\sqrt{49}-\sqrt{41}}{12-10}}$

Estimate the average rate of change by finding the slopes of each secant line. Indicate units of measure



Q1(5,20)
Q2(7,38)
Q3(8.5, 55)
Q4(9.5,71)
P(10,80)

$$\boxed{\lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

PQ1= $\frac{80-20}{10-5} = 12$ m/sec

PQ2= $\frac{80-38}{10-7} = 14$

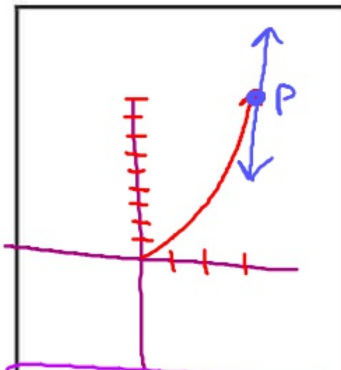
PQ3= $\frac{80-55}{10-8.5} = 16.\overline{6}$

PQ4= $\frac{80-71}{10-9.5} = 18$

Use the slopes of the secant lines to Estimate the instantaneous rate of change/slope at point P

$\cancel{\lim_{x\to a}} = \lim_{x \to P} \frac{\Delta y}{\Delta x} = \lim_{x \to P} (\text{slopes}) = 20$ ft/sec instantaneous rate of change = slope of tangent line = $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

Using a definition of the derivative to find slope



A) Find the slope of f(x) = x^2 at the point (3,9)

$f(a)=9$

$f(3)=9$

$a=3$

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$\boxed{\lim_{x\to 3}\frac{x^2-9}{x-3}=\lim_{x\to 3}x+3=6}$$

$$\frac{x^2-9}{x-3}=\frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$$

$$\boxed{\text{slope}=f'(3)=6}$$

B) Find the slope of $f(x)=\frac{2}{x}$ at x = 4

$f(x)=\frac{2}{x}$ $\quad x=4$ $\quad a=4$

$\left(4,\frac{1}{2}\right)$

$a \quad f(a)$

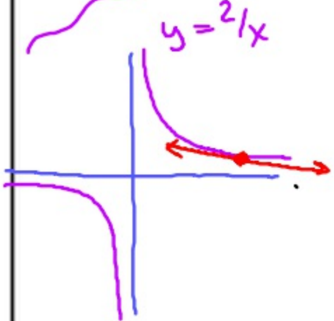
$$\boxed{f'(4)=-\frac{1}{8}}$$

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$\lim_{x\to 4}\frac{\frac{2}{x}-\frac{1}{2}}{x-4}=\lim_{x\to 4}\frac{-1}{2x}=\frac{-1}{8}$$

$$\frac{(2)}{(2)}\frac{2}{x}-\frac{1}{2}\frac{(x)}{(x)}$$

$$\frac{\left(\frac{\cancel{4-x}}{2x}\right)}{\cancel{(x-4)}}=\frac{-1}{2x}$$



C) Find the slope of $f(x)=\frac{1}{x-4}$ at x = 7

D) Find the slope of f(x) = 9 - x^2 at the point (-3,0)